SIMPLIFIED EXPRESSIONS FOR PAIR TRANSFER, ESPECIALLY N=Z NUCLEI

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In the single j shell approximation a wave function of a state of total angular momentum I in an even-even Ti isotope can be written in terms of a column vector with entries $D(J_p, J_n)$ where D is the probability amplitude that the protons couple to angular momentum J_p and the neutrons to J_n. For I = 0 which we here consider $J_p = J_n = J$. We address the problem of obtaining the number of n-p pairs in a given Ti isotope with angular momentum J_a, where J_a can range from zero to 2j. Normally, to get an expression for the number of n-p pairs we require a 9j symbol to bring together the neutron and the proton, as well as coefficients of fractional parentage to single out one neutron from the rest. However, we can with some tricks get the following simplified expressions for the members of n-p pairs for a Ti isotope with n neutrons, using any isospin conserving 2 body interaction:

For T = T_{min} = (N - Z)/2

Number of pairs($J_a = 0$) = 2 D(0,0)^2/n   Number of even pairs = n-1

For T = T_{min} + 2

Number of pairs($J_a = 0$) = 2 n D(0,0)^2 Number of even pairs = 2n

(The corresponding numbers for odd pairs are (n+1) and zero).

For an N = Z nucleus e.g. radioactive ^{44}Ti we find number of n-p pairs with even angular momentum $J_a = D(J_a, J_a)^2$ where for ^{44}Ti J_a can be 0,2,4, or 6. This is the same as the number of nn pairs and of pp pairs.

As an example for ^{44}Ti, using an interaction obtained for the spectrum of ^{42}Sc, the values of D(J,J) for J = 0,2,4 and 6 are respectively 0.7878, 0.5617, 0.2208 and 0.1234.

If we had no interaction the average number of J_a pairs for J_a = 0 to 7 would be respectively 0.750, 0.432, 0.861, 0.902, 0.750, 1.000, 0.639 and 0.667. However, with an interaction obtained from the spectrum of ^{42}Sc the corresponding values are 1.862, 0.675, 0.946, 0.271, 0.146, 0.159, 0.046, 1.895

We see that the interaction enhances the number of pairs with J_a =0,1,2 and 7 and depletes the number of pairs with J_a = 3,4,5 and 6 (total number of pairs must be 6).

Counting pairs is of course important for 2 nucleon transfer reactions, and for determining, for example if there is J = 1 T = 0 pairing in N = Z nuclei. We find the following interesting relation for T = T_{min}

$$D(00) = \frac{n}{(2j+1)} \sum_J D(JJ)(j^{n-1}jj|\}j^n J)\sqrt{(2J+1)} \tag{1}$$